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BHASKAR CLASSES PVT LTD

Inverse Trigonometric Function, Determinant & Matrices

1. Find the principal values of each of the following:
   a. $\tan^{-1}(-\sqrt{3})$
   b. $\tan^{-1}(1)$
2. Find the principal values of each of the following:
   a. $\tan^{-1}\left\{\sin\left(-\frac{\pi}{2}\right)\right\}$
   b. $\tan^{-1}\left\{\cos\frac{3\pi}{2}\right\}$
3. For the principal values, evaluate each of the following:
   a. $\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\}$
   b. $\cot[\sin^{-1}\{\cos(\tan^{-1}1)\}]$
4. For the principal values, evaluate the following:
   a. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$
   b. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - 2\sec^{-1}\left(2\tan\frac{\pi}{6}\right)$
5. Find the principal values of each of the following:
   a. $\csc^{-1}(-\sqrt{2})$
   b. $csc^{-1}(-2)$
   c. $\csc^{-1}\left(\frac{2}{\sqrt{3}}\right)$
   d. $\csc^{-1}\left(2\cos\frac{2\pi}{3}\right)$
6. Prove that: $\cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+sin\,x}-\sqrt{1-sin\,x}}\right\} = \frac{x}{2}, 0 < x < \frac{\pi}{2}$.
7. Prove that: $\cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+sin\,x}-\sqrt{1-sin\,x}}\right\} = \frac{\pi}{2} - \frac{x}{2},\ if\ \frac{\pi}{2} < x < \pi$.
8. Find the greatest and least values of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$.
9. Solve: $\sin\left\{\sin^{-1}\frac{1}{5} + \cos^{-1}x\right\} = 1$.
10. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, prove that $A - A^T$ is a skew-symmetric matrix.

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11. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 & -4 \end{bmatrix}$, verify that $(AB)^T = B^T A^T$.

12. Evaluate the following: $\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}\right)$.

13. If ω is a complex cube root of unity, show that:

$$\left(\begin{bmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{bmatrix} + \begin{bmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 \\ \omega \\ \omega^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

14. If $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $x^2 = -1$, then show that $(A+B)^2 = A^2 + B^2$.

15. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find $A^2 - 5A - 14I$. Hence, obtain A^3.

16. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show thar $|2A| = 4|A|$.

17. Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.

18. Evaluate $\Delta = \begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$.

19. Show that $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$.

20. Show that:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca).$$