BHASKAR CLASSES PVT LTD

Continuity, Differentiability and Differentiation

1. Discuss the continuity of the following functions at the indicated point(s)

$$f(x) = \begin{cases} |x| \cos\left(\frac{1}{x}\right), & x \neq 0 \\ 0 & , x = 0 \end{cases} \quad at\ x = 0$$

2. Find the value of $'a'$ for which the function f defined by

$f(x) = \begin{cases} a \sin\frac{\pi}{2}(x+1), x \leq 0 \\ \frac{\tan x - \sin x}{x^3} \quad , x > 0 \end{cases}$ is continuous at $x = 0$.

3. Discuss the continuity of $f(x) = \begin{cases} 2x - 1, x < 0 \\ 2x + 1, x \geq 0 \end{cases} at\ x = 0.$

4. Prove that the function $f(x) = \begin{cases} \frac{x}{|x|+2x^2}, x \neq 0 \\ k, x = 0 \end{cases}$ remains discontinuous at $x = 0$, regardless the choice of k.

5. Determine the values of a, b, c for which the function $f(x) =$

$$\begin{cases} \frac{\sin(a+1)x + \sin x}{x} & , for < 0 \\ c \quad , for\ x = 0\ is\ continuous\ at\ x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}} & , for\ x > 0 \end{cases}.$$

6. If $f(x) = \begin{cases} \frac{1-\cos kx}{x \sin x}, x \neq 0 \\ \frac{1}{2}, x = 0 \end{cases}$ $is\ continuous\ at\ x = 0, find\ k.$

7. If $f(x) = \begin{cases} \frac{2^{x+1}-16}{4^x-16}, if\ x \neq 2 \\ k \quad , if\ x = 2 \end{cases}$ $is\ continuous\ at\ x = 2, find\ k.$

8. Let $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & , if\ x < 0 \\ a & , if\ x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}} - 4} & , if\ x > 0 \end{cases}$

Determine the value of a so that $f(x)$ is continuous at $x = 0$.

9. If $f(x) = \frac{\sqrt{2}\cos x - 1}{\cot x - 1}, x \neq \frac{\pi}{4}$. Find the value of $f\left(\frac{\pi}{4}\right)$ so that $f(x)$ becomes continuous at $x = \pi/4$.

10. Show that $f(x) = |x|$ is not differentiable at $x = 0$.

11. Show that the function $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), if\ x \neq 0 \\ 0 \qquad\qquad , if\ x = 0 \end{cases}$ is differentiable at $x = 0\ and\ f'(0) = 0$.

12. Discuss the differentiability of $f(x) = \begin{cases} x\, e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, x \neq 0 \\ 0 \qquad\qquad , x = 0 \end{cases}$ at $x = 0$.

13. Show that $f(x) = |x - 3|$ is continuous but not differentiable at $x = 3$.

14. If f is defined by $f(x) = x^2 - 4x + 7$, show that $f'(5) = 2f'\left(\frac{7}{2}\right)$.

15. If $f(x) = |\cos x|$, find $f'\left(\frac{\pi}{4}\right)\ and\ f'\left(\frac{3\pi}{4}\right)$.

16. If $f(x) = |\cos x - \sin x|$, find $f'\left(\frac{\pi}{6}\right)$ and $f'\left(\frac{\pi}{3}\right)$.

17. Differentiate the following functions from first principles e^{3x}.

18. Differentiate the following functions with respect to x

$\log\left(x + \sqrt{a^2 + x^2}\right)$.

19. Differentiate the following functions with respect to x

$\log\left(x + 2 + \sqrt{x^2 + 4x + 1}\right)$.

20. If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$, prove that $\frac{dy}{dx} = \frac{x-1}{2x(x+1)}$.

21. If $y = \sqrt{a^2 - x^2}$, prove that $y\frac{dy}{dx} + x = 0$.

22. Differentiate the following functions with respect to x

$\sin^{-1}\left\{\frac{\sin x + \cos x}{\sqrt{2}}\right\}, -\frac{3\pi}{4} < x < \frac{\pi}{4}$.

23. If $y = \cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}$, show that $\frac{dy}{dx}$ is independent of x.

24. If $\tan^{-1}\left\{\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right\}$, find $\frac{dy}{dx}$.

25. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$, prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.